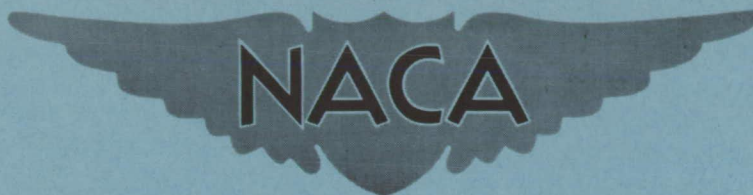


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RESEARCH MEMORANDUM

SEVERAL FACTORS AFFECTING ROLL CONTROL SYSTEMS
OF INTERCEPTORS

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON

November 6, 1953

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RESEARCH MEMORANDUM

SEVERAL FACTORS AFFECTING ROLL CONTROL SYSTEMS
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INTRODUCTION

One of the primary defense weapons of our country's air-defense system will be the manned all-weather interceptor, which is to be capable of flying at supersonic speeds and of operation to an altitude of 60,000 feet. During the attack phase of the interceptor's mission, which is started as soon as the interceptor establishes contact with the target, the radar continuously furnishes target data such as range and azimuth and elevation angles to the fire-control computer. These measured data are processed by the fire-control computer to obtain command signals to deflect the airplane's control surfaces. In order for the interceptor to maneuver toward the target, it must roll to turn. It is, therefore, necessary to incorporate an effective roll control system in the automatic guidance or tracking system of such airplanes. As part of a general investigation of the problems of automatic control of interceptors, theoretical studies of roll control systems are being conducted in the Langley stability analysis section. The purpose of this paper is to present the preliminary results of a theoretical investigation concerned with two different types of roll control systems.

DISCUSSION

Several recent design studies (ref. 1, for instance) for both longitudinal and lateral automatic-control systems have considered different types of compensating networks. The purpose of a compensating network is to cancel the effect of either one or more of the airplane's modes of motion from the response of the airplane to a command input. For example, the long-period longitudinal oscillation, the phugoid, may be canceled by a compensating network designed for an attitude control system, or the Dutch roll oscillation may be canceled by a compensating network designed for a lateral control system. A general study of compensating networks was undertaken by Windsor L. Sherman of the Langley stability analysis section and the first network studied was one designed to cancel the

effect of the airplane dynamics, that is, all the lateral modes of motion, from the response of the airplane in roll. Thus, the first type of roll control system to be discussed is referred to as a compensating network system. In order to accomplish this cancellation, the compensating network is so designed that its transfer function is the inverse of the transfer function of the airplane.

Figure 1 shows a block diagram of one type of compensating network incorporated in a roll control system. This system corresponds to a velocity command system that could control any one of a number of positional quantities such as the radar tracking error, airplane attitude, or, as in this case, controlling bank angle. An error in bank angle is immediately changed to a command in rolling velocity which passes through an integrator into the compensating network where $H(p)$ is the airplane's transfer function of rolling velocity due to an aileron deflection. The equation for $H(p)$ is given in figure 1. The output of the compensating network is fed into a hydraulic servomotor represented by a first-order time lag $1/(1+\tau p)$ and results in an aileron deflection which causes the airplane to roll. It is seen that the compensating network is so designed that its transfer function includes the inverse of the airplane's transfer function $H(p)$. As would be expected, the closed-loop transfer function ϕ_0/ϕ_1 of this system, presented in figure 1, is solely a function of the gains K_E and K_I and the time constant τ and does not include the airplane dynamics. The type of motion obtained for the aileron can be determined from the transfer function of δ_a/ϕ_1 which is a product of two factors, the first being the closed-loop transfer function and the second, the inverse transfer function of the airplane where the denominator very closely represents the characteristics of one degree of freedom of the airplane in yaw. The aileron motion would, therefore, be a damped oscillatory motion, oscillating at a frequency which is approximately the airplane's natural frequency, and the damping of the oscillation is a direct function of the damping-in-yaw derivative C_{n_r} . Also, the motion in sideslip has the same period-damping relation as the aileron motion. However, the Dutch roll oscillation in the sideslip and aileron motions could be greatly reduced by using an automatic-control system regulating sideslip and yawing velocity. Figure 2 shows the response in bank due to a step input command of 60° . The motion, corresponding to the closed-loop transfer function presented in figure 1, has good response characteristics and excellent stability.

With the compensating network just described, it appears obvious that the airplane dynamics could be eliminated from the motion in bank for a command input. There are, however, several problems of interest which were investigated. The transfer-function analysis of the system resulting in the expression shown in figure 1 assumes that the system is

linear. The question arises as to how well the system behaves if nonlinearities of the type represented by limiting the maximum control deflection and maximum rate of control deflection are taken into account. Another problem directly related to compensating networks is the possibility of incomplete compensation occurring; that is, the transfer function of the compensating network will not cancel the airplane's transfer function. Incomplete compensation may be caused by designing a compensating network based on inaccurate estimates of the stability derivatives so that the airplane's transfer function in the compensating network is not the exact inverse of the true airplane's transfer function, or incomplete compensation may be due to the fact that the airplane assumes a flight condition different from the one for which the compensating network was designed. Also, one is concerned not only with knowing how well the compensating network behaves as a command system but of equal importance one must determine the ability of the compensating network to stabilize the airplane motion if the airplane is disturbed by a gust.

Figure 3 illustrates the effect of limits in the velocity command system. Time histories of the motions in bank in response to a step input command are presented. These motions as well as all subsequent motions presented were obtained on a Reeves Electronic Analog Computer at Project Cyclone. The flight conditions correspond to an interceptor flying at 60,000 feet at $M = 2$. The upper plot shows the effect of limiting the maximum rate of control deflection $\dot{\delta}$ when the maximum control deflection δ is limited to 20° . The value of $\dot{\delta}$ for the solid-line curve is $100^\circ/\text{sec}$, whereas the dashed-line curve corresponds to a value of $\dot{\delta}$ of $40^\circ/\text{sec}$, the present requirement for powered controls. Although the rise time is not affected by reducing $\dot{\delta}$, the system becomes unstable. The lower plot in figure 3 shows the effect of limiting δ for a value of $\dot{\delta}$ of $100^\circ/\text{sec}$. The solid-line curve is for the condition of $\dot{\delta} = 100^\circ/\text{sec}$ and $\delta = 20^\circ$ and the dashed-line curve represents the case of the limited value of δ being reduced to 10° . Here again, the stability is decreased as δ is decreased. Thus, in general, a reduction in either the limiting value of $\dot{\delta}$ or δ decreases the stability of the system. If the error gain is decreased, the system becomes less critical to limiting but the response is much slower.

The other problem mentioned was the effect of small inaccuracies in the estimation of the stability derivatives in designing the compensating network, thereby resulting in incomplete cancellation of the airplane's transfer function. This problem was investigated by varying each derivative and several derivatives in combination. The results indicated that small variations of most of the derivatives had little effect on the motion. However, the estimation of the directional-stability derivative $C_{n\beta}$ was critical. Figure 4 shows the effect of inaccurately estimating $C_{n\beta}$ on the motion in bank for the velocity command system. The

solid-line curve is for the case of complete cancellation where $C_{n\beta} = 0.28$. As $C_{n\beta}$ is increased to 0.32, indicated by the dashed-line curve, a slightly divergent oscillation is introduced. However, if the actual value of $C_{n\beta}$ were less than 0.28, a motion similar to the solid-line curve is obtained. The response for $C_{n\beta} = 0.24$ is shown in figure 4. It should be noted that, if rudder control is used to maintain zero sideslip during the maneuver, the roll control system may not be sensitive to the inaccurate estimate of $C_{n\beta}$.

Calculations made for the interceptor flying at $M = 1.4$ at 60,000 feet, where the stability derivatives corresponding to this new flight condition are different from those corresponding to the designed flight condition of the compensating network, show that the motion is unstable for the velocity command system but stable for a velocity-plus-acceleration command system. However, for this velocity-plus-acceleration command system, similar results on the effect of limiting and the inaccurate estimation of $C_{n\beta}$ were obtained as indicated for the velocity command system. If a displacement command system were used, it is expected that limiting would not have as pronounced an effect as shown in figure 3.

Thus far, the results presented were confined to a compensating network as a command system. In order to investigate how well the velocity command system stabilizes the airplane motion if the airplane is disturbed by a gust, the airplane was assumed to be disturbed from trim by step inputs of either $C_l = 0.005$ or $C_n = 0.005$ which were reduced to zero at the end of 2 seconds. Figure 5 shows the results of these calculations. It is apparent from this figure that the Dutch roll oscillation does appear in the motion and, thus, the compensating network does not offer any improvement when the airplane is disturbed by a gust. The integrator in the system, which gives control proportional to the integral of the bank-angle error, reduces the steady-state error to zero. Additional calculations indicated that the response is improved for higher gains until limiting of δ and $\dot{\delta}$ takes place.

The second type of roll control system investigated by Albert A. Schy and Ordway B. Gates of the Langley stability analysis section is a conventional attitude-control system with an integrator and rate and acceleration feedback. A block diagram of the system is shown in figure 6. The analysis considered three airplanes which had the characteristics shown in figure 7. The motions presented in figure 7 are the rolling velocity due to a step deflection of the aileron for each of the three airplanes. It is noted that, for airplanes B and C, the Dutch roll oscillation appears in the rolling motion whereas, for airplane A, the motion resembles the response obtained from considering the airplane in only one degree of freedom in

roll. The corresponding sideslip motions will in all cases contain the Dutch roll characteristics. For these airplanes it was found that a yaw damper very effectively stabilized the sideslip motion but increased the steady-state sideslip angle to approximately 1° . Where the coupling between roll and sideslip is mainly a product of inertia effect, as in airplane B, a relatively larger amount of yaw damping is required to eliminate the Dutch roll from the rolling motion, thus resulting in a larger steady-state sideslip angle.

The results presented in the following figures indicate trends common to all three airplanes when equipped with a yaw damper and the roll control system outlined in the block diagram. The response of the airplane to a step-command input was very satisfactory without the integrator if the airplane has approximately neutral spiral stability. However, in analyzing the motion of the airplane after being disturbed by a gust, no satisfactory response was obtained as the gains of the system were varied; however, the integrator was required in particular to obtain zero steady-state error. Figure 8 shows the effect of varying the integrator constant K_I on the motion in bank when the airplane is disturbed from trim by a step input of $C_l = 0.01$. As K_I is increased, the airplane returns to its initial trimmed position much faster but, with further increase in K_I , the motion becomes oscillatory and will become unstable for larger values of the integrator constant. If, therefore, the integrator is an essential component of the system to obtain satisfactory response and reduces the steady-state error to zero when the airplane is disturbed by a gust, it is of interest to know the effect of the integrator on the airplane response to a step-command input. This effect is shown in figure 9. The solid-line curve in this figure represents the type of response obtained to a 60° bank-angle command without an integrator. In general, the response is excellent. With the integrator included in the system, the motion overshoots the commanded 60° bank angle and the response time, the time required for the motion to reach and remain within 5 percent of the steady-state value, is increased. This overshoot increases and the motion becomes oscillatory as the integrator gain is increased. In an attempt to improve the airplane response to a step-command input with an integrator present, the gains of the system were varied in order to obtain a more satisfactory response. First, more rate feedback was added to the system. For comparative purposes, in the lower part of figure 9 the dashed-line curve is replotted and compared with the solid-line curve which corresponds to a case which has double the rate feedback of the dashed-line curve. It is noted that the peak overshoot is not reduced and the time required for the motion to reach steady state is increased. The forward-loop gain was then varied. Figure 10 shows that a marked improvement in the command response can be realized by increasing the forward-loop gain. As the gain is increased, the overshoot is eliminated and satisfactory response is obtained. However, further increase in K_F causes the airplane to respond faster but once again introduces overshoot. This overshoot due to the

forward-loop gain results from insufficient damping in the roll oscillation and can be eliminated by rate feedback. The lower part of figure 10 shows this effect. Thus, from a linear analysis of the problem, one can determine combinations of the gains in the system which will result in a very satisfactory response in bank.

The importance of taking into account the limits on control deflection and rate of control-surface deflection is shown in figure 11. As an example, the case which resulted in a satisfactory response in bank that was based on the linear analysis was selected. The solid-line curve corresponds to the case where δ is limited to 20° and $\dot{\delta}$ is limited to $120^\circ/\text{sec}$. If the maximum rate of control deflection is reduced to $40^\circ/\text{sec}$, the motion becomes oscillatory with a large amount of overshoot, although the rise time is not seriously affected. If the limited value of δ is reduced to 5° , the stability is improved but the rise time is much slower. By knowing the maximum values of δ and $\dot{\delta}$ for which the airplane and hydraulic servomotor are designed, the values of gain constants could be selected to give good response in bank. For example, with limits on δ of 20° and on $\dot{\delta}$ of $40^\circ/\text{sec}$, the motion may be stabilized by introducing additional rate feedback into the system (see lower part of fig. 11).

In figure 11, comparison of the solid-line curve in the top plot with the solid-line curve in the lower plot indicates that the rise time for the system with a lower limited value of $\dot{\delta}$ and the proper gain values is slightly longer than for the higher limited value of $\dot{\delta}$ and the corresponding combination of gains for an input command of 60° bank angle. However, for smaller input command signals, the rise time for lower $\dot{\delta}$ is appreciably longer than the rise time obtained for higher $\dot{\delta}$. The reason is that for smaller inputs little limiting takes place and, thus, the system optimized for lower $\dot{\delta}$ has too much damping which tends to slow up the response.

Where limiting causes a lightly damped oscillation in the airplane motion, introducing acceleration feedback may have a pronounced stabilizing effect, particularly for airplanes that have a relatively low moment of inertia in roll.

CONCLUDING REMARKS

The results of the preliminary study indicate that, for both roll control systems investigated, satisfactory response to command inputs is obtained, provided the rate and physical limits of control deflection are high. By taking account of the maximum value of control displacement and control rate, the system may be optimized by proper selection of the gains. However, the motion will be faster for all magnitudes of input command signals as the values of the limits are increased.

If the airplane is disturbed by a gust, the integrator present in both systems reduces the steady-state error to zero but the compensating network does not cancel the airplane dynamics from the response.

With a compensating network system, incomplete compensation will result if the estimated value of $C_{n\beta}$ used in designing the network is less than the actual $C_{n\beta}$ value of the airplane. However, if rudder control is used to maintain zero sideslip during the maneuver, the roll control system may not be sensitive to the inaccurate estimate of $C_{n\beta}$.

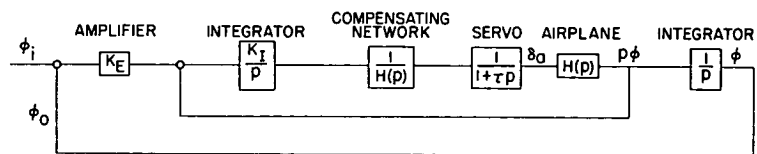
With a conventional attitude-roll control system, the Dutch roll oscillation present in the sideslip motion is effectively stabilized through use of a yaw damper but the steady sideslip angle is increased.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 1, 1953.

REFERENCE

1. Owen, J. C.: Report on Automatic Pilot for High Performance Aircraft. U.S.A.F. Exhibit MCREXell-133, Contract AF-33(038)5700. Rep. No. 8, Eclipse-Pioneer Div. of Bendix Aviation Corp., Sept.-Oct. 1950, pp. 35-50.

BLOCK DIAGRAM FOR COMPENSATING NETWORK SYSTEM



$$H(p) = \frac{p(a_2 p^2 + a_1 p + a_0)}{b_4 p^4 + b_3 p^3 + b_2 p^2 + b_1 p + b_0}$$

$$\frac{\phi_o}{\phi_i} = \frac{K_E K_I}{\tau p^3 + p^2 + K_I p + K_E K_I}$$

$$\frac{\delta_a}{\phi_i} = \left(\frac{\phi_o}{\phi_i} \right) \left(\frac{p}{H(p)} \right)$$



Figure 1

RESPONSE IN BANK FOR STEP INPUT COMMAND

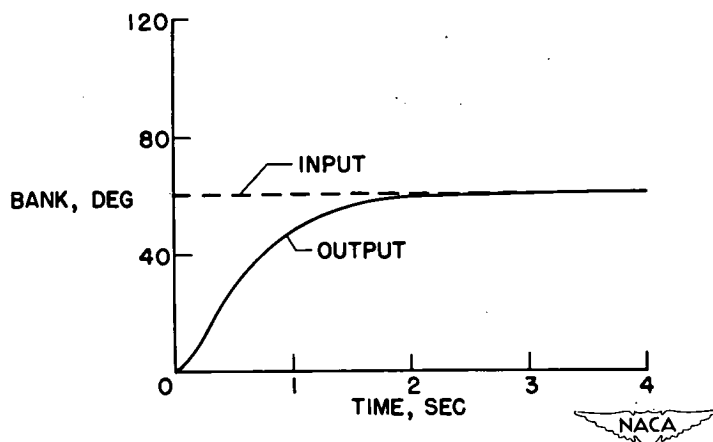


Figure 2

EFFECT OF LIMITING CONTROL DEFLECTION AND RATE OF CONTROL DEFLECTION

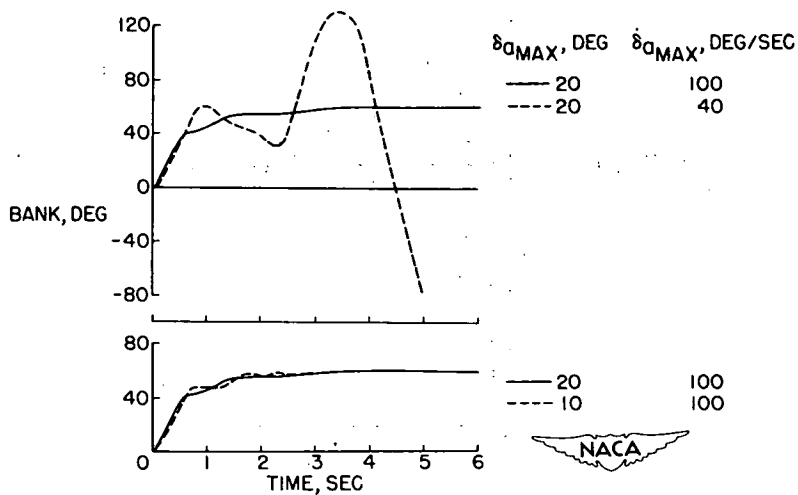


Figure 3

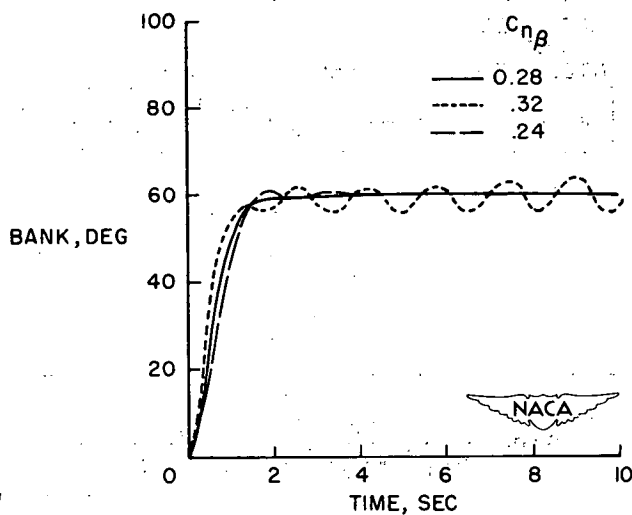
EFFECT OF INACCURACY IN ESTIMATED $C_{n\beta}$ 

Figure 4

GUST RESPONSE OF AIRPLANE WITH COMPENSATING NETWORK

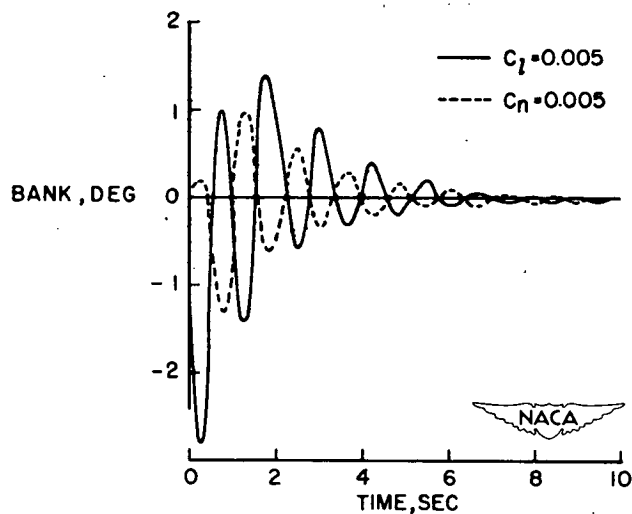
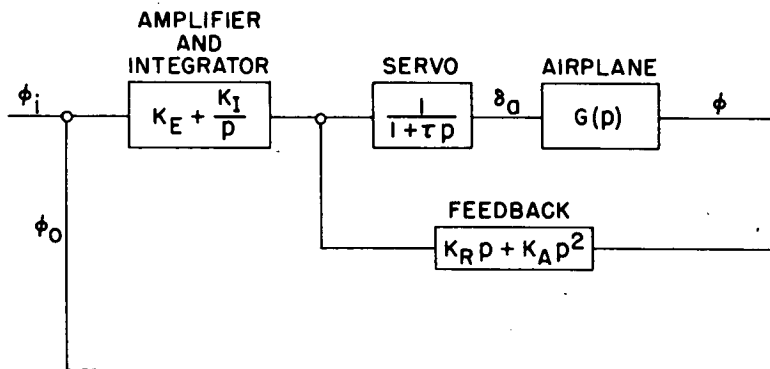


Figure 5

BLOCK DIAGRAM OF AN ATTITUDE CONTROL SYSTEM



$$G(p) = \frac{a_2 p^2 + a_1 p + a_0}{b_4 p^4 + b_3 p^3 + b_2 p^2 + b_1 p + b_0}$$



Figure 6

STABILITY CHARACTERISTICS OF AIRPLANES INCLUDED IN ANALYSIS

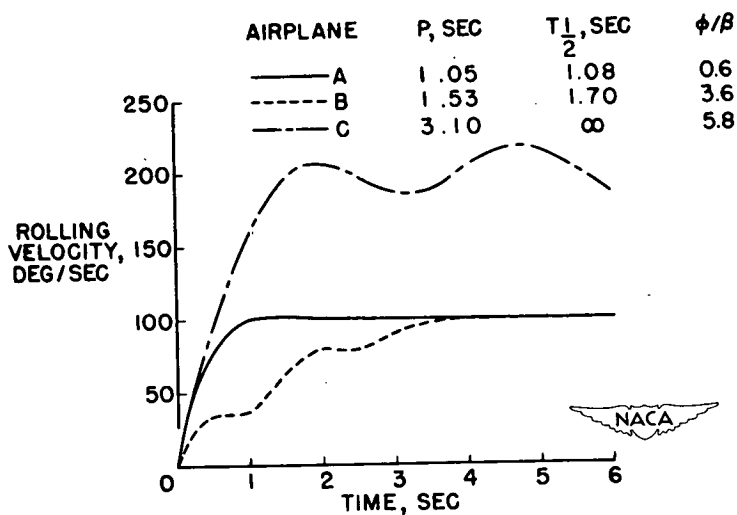


Figure 7

EFFECT OF INTEGRATOR ON REGULATORY RESPONSE

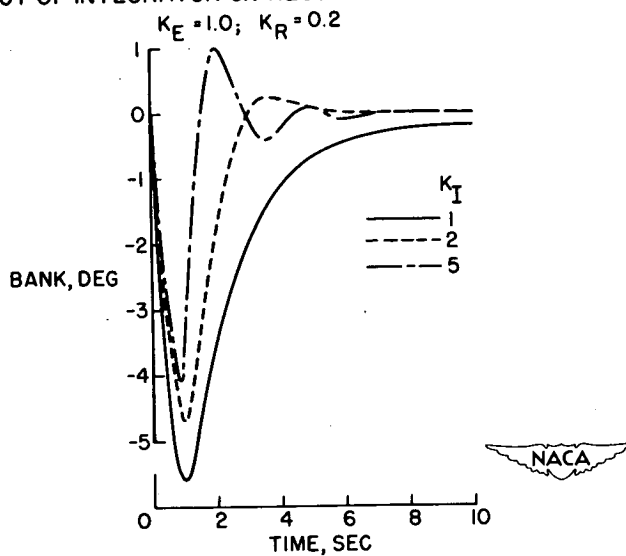


Figure 8

EFFECT OF INTEGRATOR AND RATE FEEDBACK ON COMMAND RESPONSE

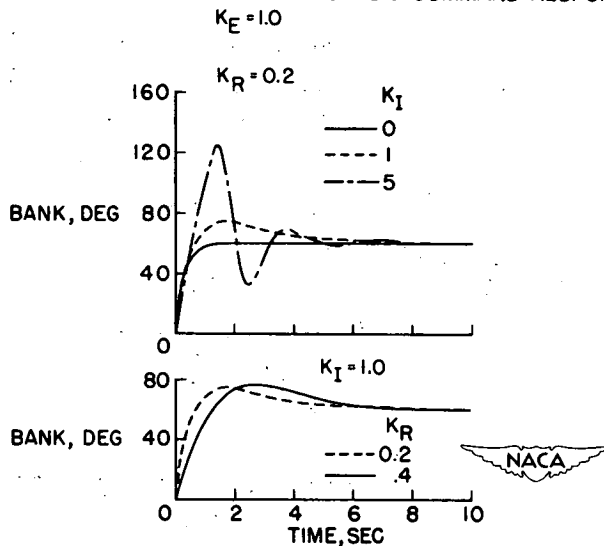


Figure 9

EFFECT OF FORWARD-LOOP GAIN ON COMMAND RESPONSE

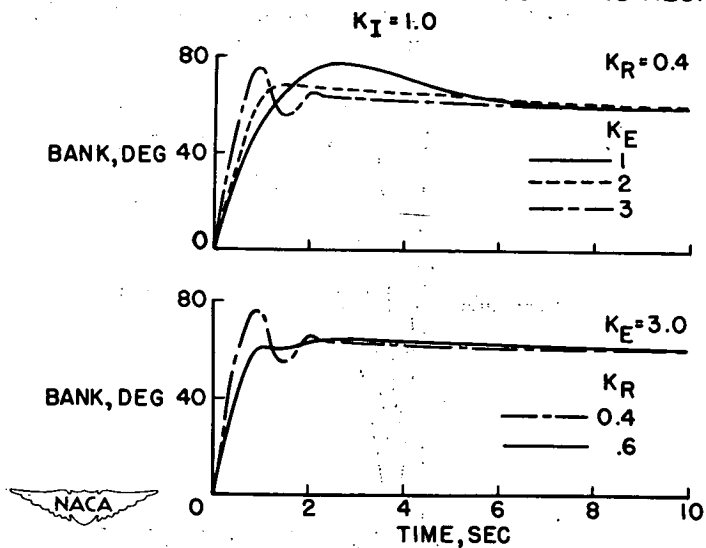


Figure 10

EFFECT OF LIMITING CONTROL DEFLECTION AND
RATE OF CONTROL DEFLECTION

$$K_E = 3.0; K_I = 1.0$$

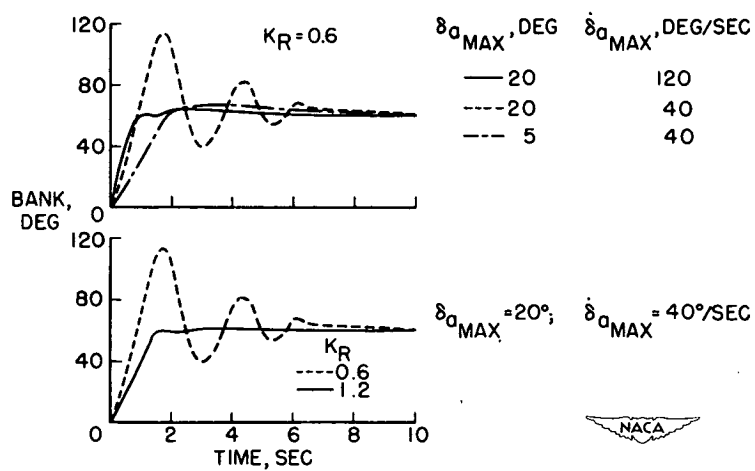


Figure 11